viernes, 29 de mayo de 2020 2:17

Universidad de Tarapacá
Facultad de Ciencias
Dpto. de Matemática

TUTORIA N°3
CÁLCULO 1

1. Determinar el dominio y recorrido de las siguientes funciones:

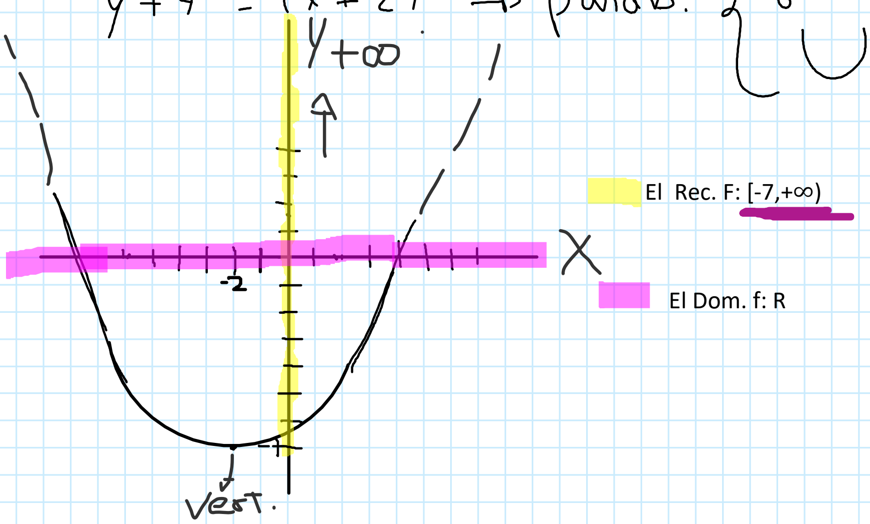
$a)\ f(x) = x^2 + 4x - 3$

$y = x^2 + 4x - 3$

$y + 3 = x^2 + 4x + 4 - 4$

$y + 7 = (x+2)^2 \rightarrow$ parab.

$\begin{cases} C\ (-2, -7) \\ \text{eje F} \parallel \text{eje } y \\ \cup \rightarrow \text{Parab.} \end{cases}$



El Rec. F: [-7,+∞)

El Dom. f: R

Otra forma de calcular el dominio:

Dom f(x) = x ϵ R : f(x)ϵR

= R : $x^2 + 4x - 3\ \epsilon\ R$

= R : $(x-2)^2 - 7\ \ \forall_x\ \epsilon\ R$

Dom f = R

$b)\ f(x) = \sqrt{(x-1)(x-9)}$

$y = \sqrt{(x-1)(x-9)}\ \ /(\ )^2$

$y^2 = (x-1)(x-9)$

$y^2 = x^2 - 9x - x + 9$

$y^2 - x^2 + 10x = 9$

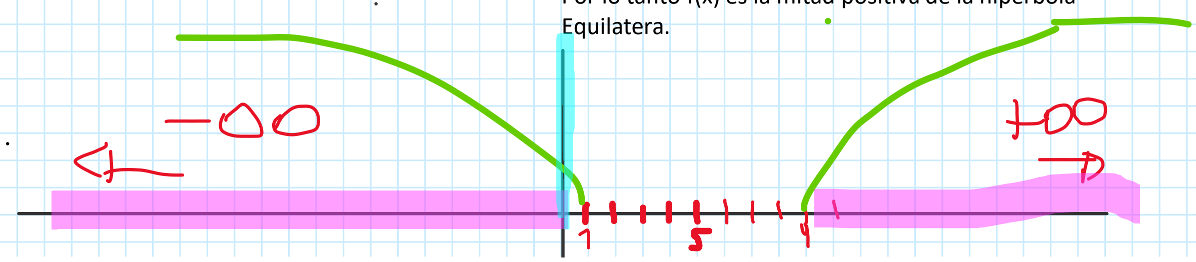
$y^2 - (x^2 - 10x + 25 - 25) = 9$

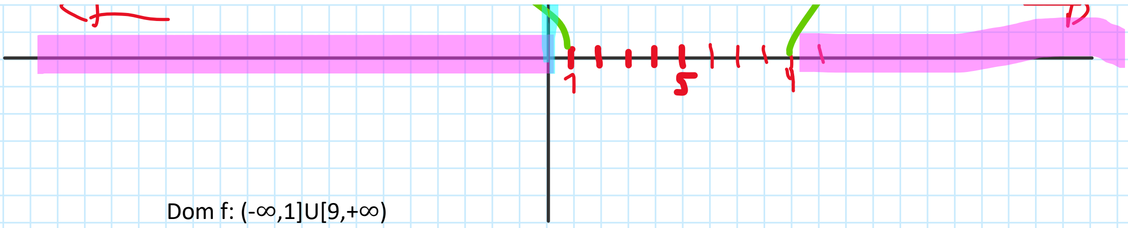
$y^2 - (x-5)^2 = 9 - 25$

$y^2 - (x-5)^2 = -16\ \ / -16$

$\left.\begin{array}{l} \dfrac{(x-5)^2}{16} - \dfrac{y^2}{16} = 1 \\ a = b = 4 \end{array}\right\}$

Hiperbola equilatera
Centro(5.0)
a=b=4
Por lo tanto f(x) es la mitad positiva de la hiperbola Equilatera.





Dom f: (-∞,1]U[9,+∞)
Rec f: [0,+∞) o tambien se puede escribir como $R^{+}U\ \{0\}$

C) $g(x) = \sqrt{1-x}$

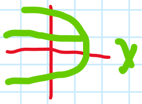
$(y-k)^2 = 4p(x-h)$

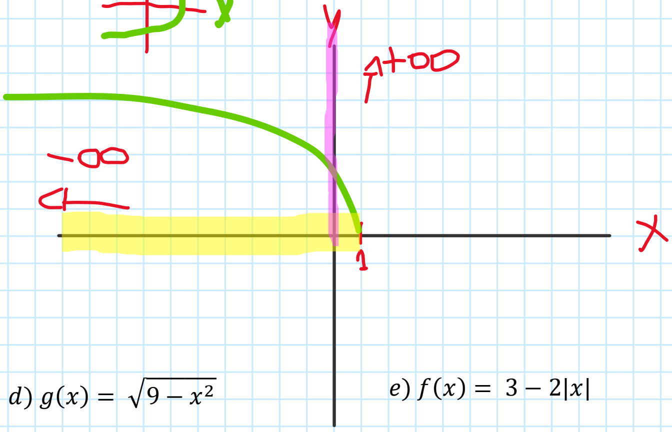
$y = \sqrt{1-x} \quad /()^2$

$y^2 = 1-x$

$y^2 = -(x-1)$

Mitad de una parabola
Eje focal // eje x
Centro (1,0)





Dom g: (-∞,1]

Rec g: [0,+∞)

d) $g(x) = \sqrt{9-x^2}$ e) $f(x) = 3 - 2|x|$

1. **(Ejercicio Control).** Sea la función f definida por:

$$f(x): \begin{cases} \text{i)}\ 4+\sqrt{9\left(1-\frac{(x+10)^2}{25}\right)}, & x\in[-10,-5) \longrightarrow f_1 \\ \text{ii)}\ 4-\frac{(x-2)^2}{2}, & x\in(-2,2) \longrightarrow f_2 \\ \text{iii)}\ 3-\sqrt{9\left(1+\frac{(x-4)^2}{16}\right)}, & x\in(4,+\infty) \longrightarrow f_3 \end{cases}$$

Determinar: a) La grafica de f

b) Dominio de f

b) Recorrido de f

i) Función por tramo
ii) Dom f= dom f1 U Domf2 U Dom f3
= [-10,-5)U(-2,2)U(4,+∞)

i) $y = 4 + \sqrt{9\left(1 - \frac{(x+10)^2}{25}\right)}$

$y-4 = \sqrt{9\left(1-\frac{(x+10)^2}{25}\right)} \quad /()^2$

$(y-4)^2 = 9\left(1-\frac{(x+10)^2}{25}\right)$

$\frac{(y-4)^2}{9} = 1 - \frac{(x+10)^2}{25}$

$$\frac{(y-4)^2}{9} = 1 - \frac{(x+10)^2}{25}$$

$$\frac{(x+10)^2}{25} + \frac{(y-4)^2}{9} = 1 \longrightarrow$$

$25 \to a^2$, $9 \to b^2$

Ecc. Elipse con centro (-10,4)
a=5, b=3
Elipse vertical
Eje focal //eje x

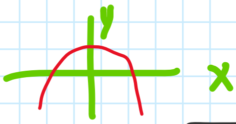
ii) $y = 4 - \frac{(x-2)^2}{2}$

$$y - 4 = -\frac{(x-2)^2}{2}$$

$$2(y-4) = -(x-2)^2 \quad / \cdot -1$$

$$(x-2)^2 = -2(y-4) \longrightarrow$$

Ecc. Parabala
Centro (2,4)
Eje focal // eje y



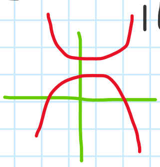
iii) $y = 3 - \sqrt{9\left(1+\frac{(x-4)^2}{16}\right)} \quad /(\ )^2$

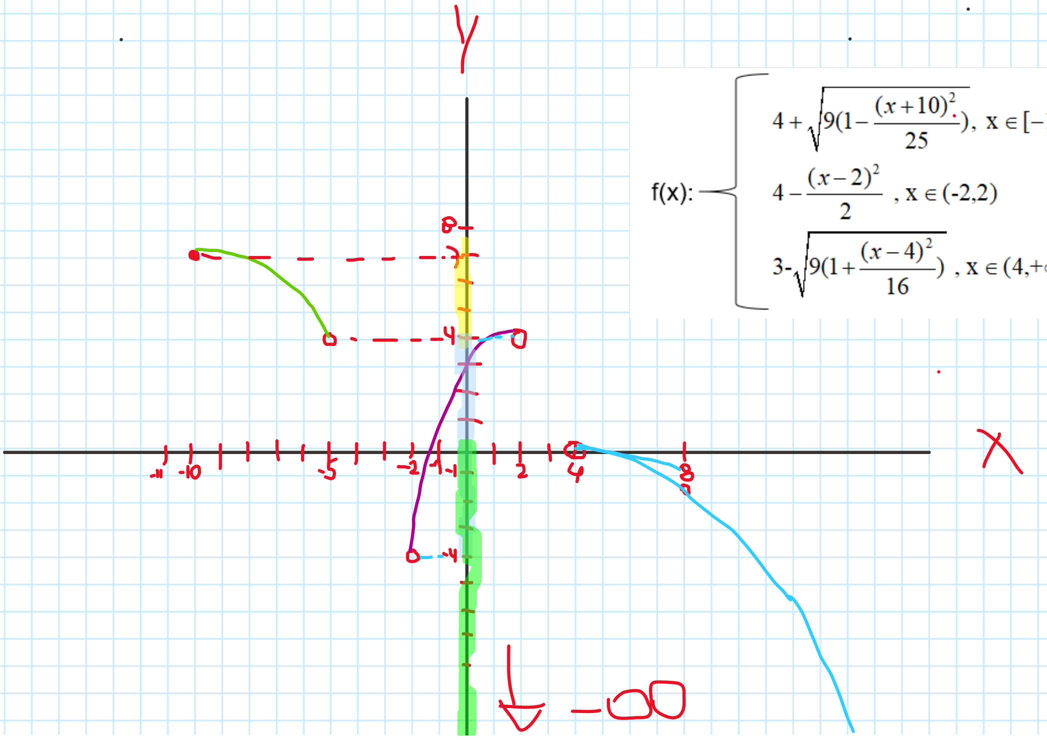
$$(y-3)^2 = 9\left(1+\frac{(x-4)^2}{16}\right)$$

$$\frac{(y-3)^2}{9} = 1 + \frac{(x-4)^2}{16}$$

$$\frac{(y-3)^2}{9} - \frac{(x-4)^2}{16} = 1 \longrightarrow$$

Ecc. Hiperbola
Centro(4,3)
Hiperb. Vertical
a^2=9, b^2=16





$$f(x): \begin{cases} 4+\sqrt{9\left(1-\frac{(x+10)^2}{25}\right)}, & x \in [-10,-5) \\ 4-\frac{(x-2)^2}{2}, & x \in (-2,2) \\ 3-\sqrt{9\left(1+\frac{(x-4)^2}{16}\right)}, & x \in (4,+\infty) \end{cases}$$

i) Elipse $x \in [-10,-5)$

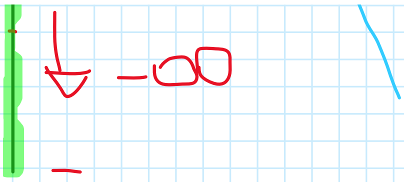
x	y
-10	7
-5	4

ii) Parábola $x \in (-2,2)$

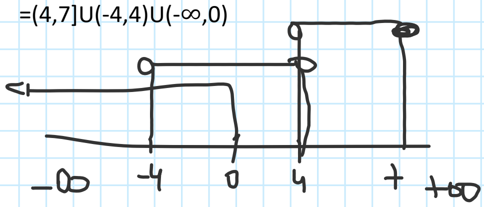
x	y
-2	-4
2	4

iii)Hiperbola $x \in (4,+\infty)$

x	y
4	0
8	$3-3\sqrt{2} \approx -1{,}24$



Rec f: rec f1 U Recf2 U Recf3
=(4,7]U(-4,4)U(-∞,0)



Rec f: (-∞,4)U(4,7)