viernes, 26 de junio de 2020 3:24

Universidad de Tarapacá
Facultad de Ciencias
Dpto. de Matemática

TUTORIA N°6
CÁLCULO 1

1. Sean $f(x) = \frac{\log(x-1)-2x}{3}$, $si\ x > 1$ y $g(x)\begin{cases} 10^x, x \leq 2 \\ \sqrt{x+3}, 2 < x < 7 \end{cases}$, Determinar $(f+g) y\ (fog)$

i) $(f+g)(x) = f(x) + g(x)$

$\Rightarrow \mathrm{Dom}(f+g) = \mathrm{Dom} f \cap \mathrm{Dom} g$

$\mathrm{Dom} f = (1, +\infty)$
$\mathrm{Dom} g_1 = (-\infty, 2]$

$\mathrm{Dom}(f+g_1) = (1, +\infty) \cap (-\infty, 2]$
$= (1, 2]$

$(f+g_1)(x) = f(x) + g_1(x)$
$= \frac{\log(x-1)-2x}{3} + 10^x$

$(f+g_2)(x) = \frac{\log(x-1)-2x}{3} + \sqrt{x+3}$

$\mathrm{Dom}(f+g_2) = \mathrm{Dom} f \cap \mathrm{Dom} g_2$
$= (1, +\infty) \cap (2, 7)$

$\mathrm{Dom}(f+g_2) = (2, 7)$

$$\therefore (f+g)(x) \begin{cases} \frac{\log(x-1)-2x}{3} + 10^x, & x \in (1, 2] \\ \frac{\log(x-1)-2x}{2} + \sqrt{x+3}, & x \in (2, 7) \end{cases}$$

ii) $fog \rightarrow fog_1$ y fog_2

ii) $f \circ g \rightarrow f \circ g_1$ y $f \circ g_2$

$(f \circ g)$ $\quad$ $\mathrm{Dom}\, f\circ g \neq \emptyset$

1) $\mathrm{Dom}(f\circ g_1)(x) = \{x \in \mathrm{Dom}\, g_1 \wedge g_1 \in \mathrm{Dom}\, f\}$

$= \{x \in (-\infty, 2] \wedge 10^x \in (1, +\infty)\}$

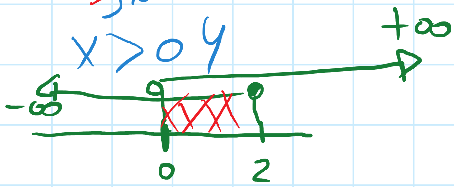
$= \{x \in (-\infty, 2] \wedge 10^x > 1 \ /\log\}$

$= \{x \in (-\infty, 2] \wedge \log 10^x > \log 1\}$

$= \{x \in (-\infty, 2] \wedge x \cancel{\log_{10} 10}^{\,1} > 0\}$

$= \{x \in (-\infty, 2] \wedge x > 0\}$

$\mathrm{Dom}(f\circ g) = (0, 2]$



Prop de log

$\log_{10} 10 = 1$

$\log_a a = 1$

$\log 10^2 = 2 \log 10$

y $(f\circ g)(x) = f(g_1(x)) = f(10^x)$

$(f\circ g_1) = \dfrac{\log(10^x - 1) - 2\cdot 10^x}{3}$ $\quad x \in (0, 2]$

2) $\mathrm{Dom}(f\circ g_2)(x) = \{x \in \mathrm{Dom}\, g_2 \wedge g_2 \in \mathrm{Dom}\, f\}$

$= \{x \in (2, 7) \wedge \sqrt{x+3} \in (1, +\infty)\}$

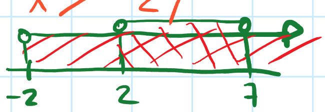
$= \{x \in (2, 7) \wedge \sqrt{x+3} > 1\}$ $\quad [\sqrt{\ }]^2 = |\ |$

$= \{x \in (2, 7) \wedge |x+3| > 1\}$

$= \{x \in (2, 7) \wedge x+3 > 1 \ /-3\}$

$= \{x \in (2, 7) \wedge x > -2\}$

$\mathrm{Dom}(\circ g_2 = (2, 7)$



$\therefore\ \exists (f\circ g_2)(x) = f(g_2(x))$

$= \dfrac{\log(\sqrt{x+3} - 1) - 2\sqrt{x+3}}{3}$, $x \in (2, 7)$

$\therefore\ f\circ g \begin{cases} \dfrac{\log(10^x\ 1) - 2\cdot 10^x}{3}, & x \in (0, 2] \\ \dfrac{\log(\sqrt{x+3}\quad)\ 2\sqrt{x+3}}{3} & x \in (2, 7) \end{cases}$

2. Determinar si las funciones son par o impar

a) $g(x) = \frac{x(x+1)}{x-1}$

f: Par
$f(-x) = f(x)$

f: Impar
$f(-x) = -f(x)$

$f(-x) = f(x)$ $f(-x) = -f(x)$

a) $g(x) = \frac{x(x+1)}{x-1}$

Par: $g(-x) = g(x)$

$g(-x) = \frac{-x(-x+1)}{-x-1}$

$= \frac{-x(-x+1)}{-(x+1)}$

$= \frac{x(-x+1)}{(x+1)}$

$\therefore f(-x) \neq f(x)$, $f(x)$ no es par.

Impar: $g(-x) = -g(x)$

$= \frac{-x(-x+1)}{-x-1}$

$= \frac{x(x-1)}{-x-1}$

$\therefore f(-x) \neq -f(x)$ $f(x)$ no es impar

b) $f(x) = \frac{3}{1-x^2}$

Par: $f(-x) = f(x)$

$= \frac{3}{1-(-x)^2}$

$= \frac{3}{\underbrace{1-(x)^2}_{f(x)}}$ $\Rightarrow f(-x) = f(x)$, $\therefore f(x)$ es par.

c) $f(x) = \ln\left|\frac{1-x}{1+x}\right|$

Par: $f(-x) = f(x)$

Impar: $f(-x) = -f(x)$

$= \ln\left|\frac{1-(-x)}{1+(-x)}\right|$

$= \ln\left|\frac{1+x}{1-x}\right|$

$= \ln\left|\left(\frac{1-x}{1+x}\right)^{-1}\right|$

$= \ln\left(\left|\frac{1-x}{1+x}\right|\right)^{-1}$

$\left(\frac{3}{1}\right)^{-1} = \left(\frac{1}{3}\right)^{2}$

$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^{+m}$

$$= -1 \cdot \ln\left|\frac{1-x}{1+x}\right| = -\underbrace{\ln\left|\frac{1-x}{1+x}\right|}_{f(x)}$$

$\therefore$ f(x) es impar

$$f(-x) = -f(x)$$

d) $f(x) = |x-2| - |x+2|$ **(TAREA)**

3. Encontrar su recorrido y Analizar su paridad **(TAREA)**

$f(x) = |x^2 - 1| - |2x - 4| - x^2, si\ 0 \leq x \leq 4$