viernes, 12 de junio de 2020 11:59

Universidad de Tarapacá
Facultad de Ciencias
Dpto. de Matemática

TUTORIA N°4
CÁLCULO 1

1. Determinar el dominio y recorrido de las siguientes funciones

a)Sea $f(x) = \begin{cases} x^2, si\ x < 0 & \to f_1 \\ 1 + x, si\ 0 \le x \le 1 & \to f_2 \end{cases}$

$\text{Dom } f(x) = \text{Dom } f_1 \cup \text{Dom} f_2$
$= (-\infty, 0) \cup [0, 1]$

$\text{Dom} f(x) = (-\infty, 1]$

Recorrido de f_1 y f_2 $\to$ $\text{Rec} f(x) = \text{Rec} f_1 \cup \text{Rec} f_2$

a) $\text{Rec} f_1 = f_1 \in \mathbb{R} : x \in \text{Dom} f_1$
$= \mathbb{R} : x \in (-\infty, 0)$
$= \mathbb{R} : x < 0 \ /(\ )^2$
$= \mathbb{R} : x^2 > 0 = \mathbb{R} \cap (0, +\infty)$

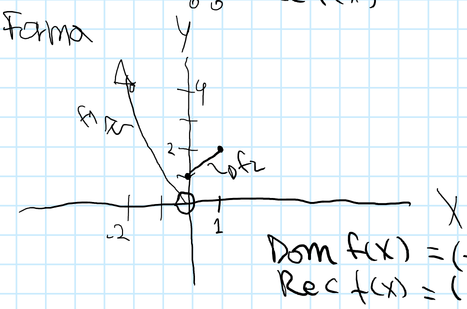
$\text{Rec} f_1 = \mathbb{R}^+ - \{0\} = \mathbb{R}^+$

b) $\text{Rec} f_2 = f_2 \in \mathbb{R} : x \in \text{Dom} f_2$
$= \mathbb{R} : x \in [0, 1]$
$= \mathbb{R} : 0 \le x \le 1 \ /+1$
$= \mathbb{R} : 1 \le \underbrace{x+1}_{f_2} \le 2$
$= \mathbb{R} \cap [1, 2]$

$\text{Rec} f_2 = [1, 2]$

$\therefore \text{Rec} f(x) = \mathbb{R}^+ \cup [1, 2] = \mathbb{R}^+$

2da Forma



f_1:

x	$f_1(x) = x^2$
0	0
-1	1
-2	4
⋮	

f_2:

x	$f_2(x) = 1 + x$
0	1
1	2

$\text{Dom } f(x) = (-\infty, 0) \cup [0, 1] = (-\infty, 1]$
$\text{Rec } f(x) = (0, +\infty) \cup [1, 2] = \mathbb{R}^+$

2. **(Ej. de Prueba)** Determinar el Dominio de:

$$f(x) = \underbrace{\sqrt{\frac{(x^2+4)(x^2-4)}{x}}}_{f'(x)} + \underbrace{\log(2x - x^2 + 3)}_{p(x)}$$

$f(x) = f'(x) + p(x)$; suma de func.

$\text{Dom } f(x) = \text{Dom } f'(x) \cap \text{Dom } p(x)$ ⊛

$\text{Dom } f'(x) = x \in \mathbb{R} : f'(x) \in \mathbb{R}$

$\text{Dom } f'(x) = x \in \mathbb{R} : f'(x) \in \mathbb{R}$

$= \mathbb{R} : \sqrt{\dfrac{(x^2+4)(x^2-4)}{x}} \in \mathbb{R}$

$= \mathbb{R} : \dfrac{(x^2+4)(x^2-4)}{x} \geq 0$

$= \mathbb{R} : (x^2+4)\ \forall_x \in \mathbb{R},\ \dfrac{x^2-4}{x} \geq 0$

Tabla Pt. Critic.

	$-\infty$ a -2	-2 a 0	0 a 2	2 a $+\infty$
$x-2$	$-$	$-$	$-$	$+$
$x+2$	$-$	$+$	$+$	$+$
x	$-$	$-$	$+$	$+$
$\dfrac{x^2-4}{x}$	$-$	$+$	$-$	$+$

$[-2,0) \cup [2,+\infty)$

$\therefore = \mathbb{R} \cap [-2,0) \cup [2,+\infty)$

$\text{Dom } f'(x) = [-2,0) \cup (2,+\infty)$

$\text{Dom } p(x) = x \in \mathbb{R} : p(x) \in \mathbb{R}$

$= \mathbb{R} : \log(2x - x^2 + 3) \in \mathbb{R}$

$= \mathbb{R} : 2x - x^2 + 3 > 0$

$= \mathbb{R} : -(x^2 - 2x - 3) > 0 \ /\cdot(-1)$

$= \mathbb{R} : x^2 - 2x - 3 < 0$

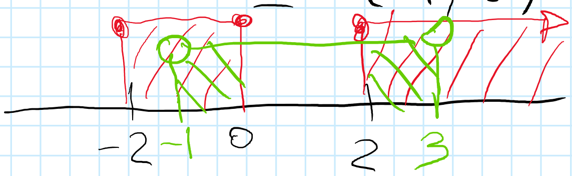
$= \mathbb{R} : (x-3)(x+1) < 0 \rightarrow$

	$-\infty$ a -1	-1 a 3	3 a $+\infty$
$x-3$	$-$	$-$	$+$
$x+1$	$-$	$+$	$+$
$(x-3)(x+1)$	$+$	$-$	$+$

$\text{Dom } p(x) = \mathbb{R} \cap (-1,3)$
$= (-1,3)$

$\therefore \text{Dom } f(x) = \{[-2,0) \cup [2,+\infty)\} \cap (-1,3)$

$= (-1,0) \cup [2,3)$



3. **(Ej. de Control)** Determinar el recorrido de:

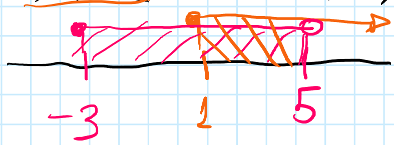
$f(x) = x^2 - 2|x-1| + 1, x \in [-3,5)$

i) $\text{Dom } f(x) = [-3,5)$

ii) Abrir el V.A

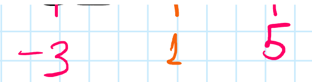
$|x-1| = \begin{cases} x-1, & x \geq 1 \quad ① \\ -(x-1), & x < 1 \quad ② \end{cases}$

① $x \geq 1 \wedge [-3,5)$, $f_1(x) = x^2 - 2(x-1) + 1$



$f_1(x) = x^2 - 2x + 2 + 1$

$f_1(x) = x^2 - 2x + 3,\ x \in [1,5)$

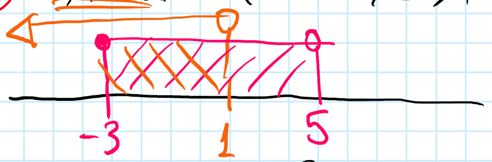


$f_1(x) = x^2-2x+3 \;,\; x\in[1,5)$

② $x<1 \wedge [-3,5),\;$ $f_2(x) = x^2-2\cdot[-(x-1)]+1$

$f_2(x) = x^2+2x-2+1$

$f_2(x) = x^2+2x-1,\; x\in[-3,1)$



$$\therefore\; f(x)\begin{cases} x^2-2x+3 \;,\; x\in[1,5) \\ x^2+2x-1 \;,\; x\in[-3,1)\end{cases}$$

Compl. de Cuadrado

$$f(x)\begin{cases} (x^2-2x+1)-1+3=(x-1)^2+2 \;,\; x\in[1,5) \\ (x^2+2x+1)-1-1=(x+1)^2-2 \;,\; x\in[-3,1)\end{cases}$$

f_1 y f_2 son parabolas donde: f_1 tiene centro $(1,2)$

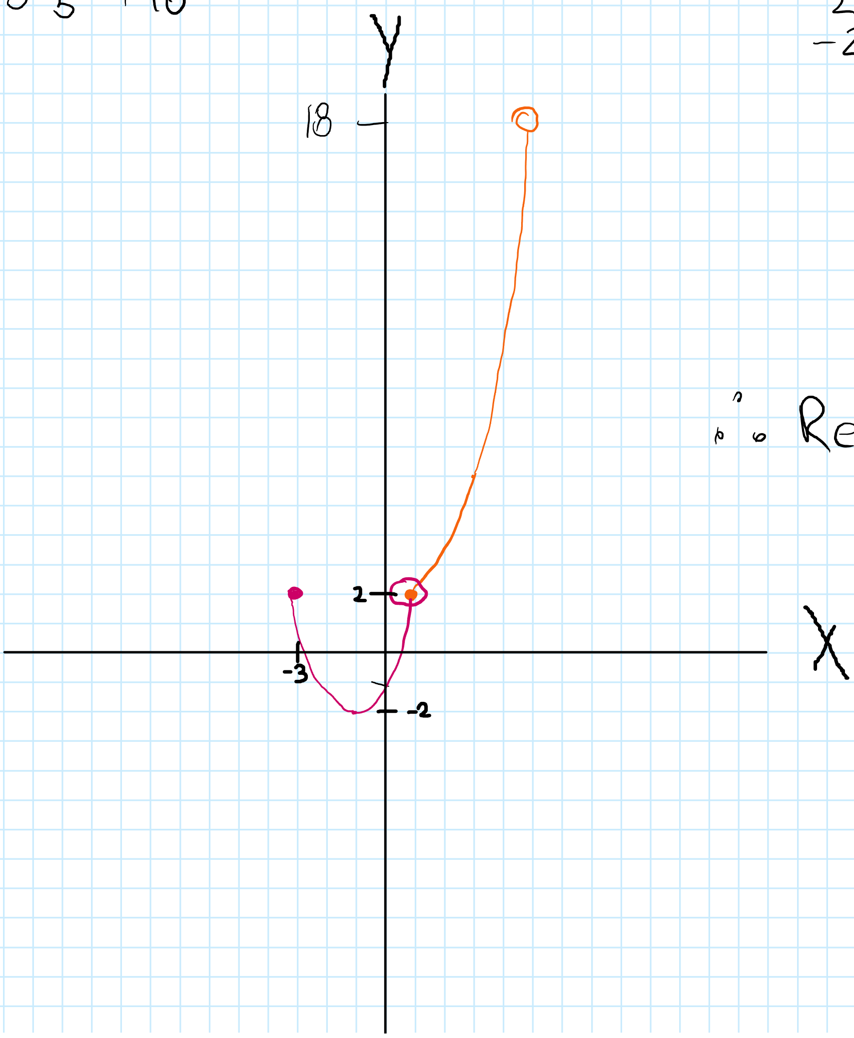
f_2 " " $(-1,-2)$

f_1:

x	$f_1(x)=(x-1)^2+2$
• 1	2
3	6
∘ 5	18

f_2:

x	$f_2(x)=(x+1)^2-2$
• -3	2
0	-1
∘ 1	2
-2	-1



$\therefore\;$ Rec $f(x) = [-2,2)\cup[2,18)$

Rec $f(x) = [-2,18)$