Universidad de Tarapacá
Facultad de Ciencias
Dpto. de Matemática

TUTORIA N°11
CÁLCULO 1

TABLA DE DERIVADAS (1)

Funciones Simples

Función	Derivada
$y = k$	$y' = 0$
$y = x$	$y' = 1$
$y = kx$	$y' = k$
$y = \frac{x}{k}$	$y' = \frac{1}{k}$
$y = \frac{k}{x}$	$y' = -\frac{k}{x^2}$
$y = x^n$	$y' = nx^{n-1}$
$y = x^{-n}$	$y' = -\frac{n}{x^{n+1}}$
$y = \sqrt{x}$	$y' = \frac{1}{2\sqrt{x}}$
$y = a^x$	$y' = a^x \cdot \ln a$
$y = e^x$	$y' = e^x$

FUNCIONES ALGEBRAICAS

Función	Derivada
$y = u \pm v \pm \ldots$	$y' = u' \pm v' \pm \ldots$
$y = k \cdot u$	$y' = k \cdot u'$
$y = u \cdot v$	$y' = u' \cdot v + u \cdot v'$
$y = u \cdot v \cdot \ldots$	$y' = u' \cdot v \cdot \ldots + u \cdot v' \cdot \ldots + \ldots$
$y = \frac{u}{v}$	$y' = \frac{u' \cdot v - u \cdot v'}{v^2}$
$y = \frac{k}{v}$	$y' = \frac{-k \cdot v'}{v^2}$
$y = \frac{u}{k}$	$y' = \frac{u'}{k}$

FUNCIONES POTENCIALES, EXPONENCIALES y V.A.

Función	Derivada
$y = u^n$	$y' = n \cdot u^{n-1} \cdot u'$
$y = u^{-n}$	$y' = \frac{-n \cdot u'}{u^{n+1}}$
$y = \sqrt[n]{u} = u^{\frac{1}{n}}$	$y' = \frac{u'}{n\sqrt[n]{u^{n-1}}} = \frac{1}{n} \cdot u^{\frac{1}{n}-1} \cdot u'$
$y = a^u$	$y' = a^u \cdot u' \cdot \ln a$
$y = e^u$	$y' = e^u \cdot u'$
$y = u^v$	$y' = v \cdot u^{v-1} \cdot u' + u^v \cdot v' \cdot \ln u$
$y = \lvert u \rvert$	$y' = \frac{u}{\lvert u \rvert} \cdot u'$

FUNCIONES LOGARÍTMICAS

Función	Derivada
$y = \log_a u$	$y' = \frac{u'}{u \cdot \ln a} = \frac{u'}{u} \cdot \log_a e$
$y = \ln u$	$y' = \frac{u'}{u}$

FUNCIONES CIRCULARES

Función	Derivada
$y = \operatorname{sen} u$	$y' = u' \cdot \cos u$
$y = \cos u$	$y' = -u' \cdot \operatorname{sen} u$
$y = \operatorname{tg} u$	$y' = u' \cdot \sec^2 u$
$y = \operatorname{cotg} u$	$y' = -u' \cdot \operatorname{cosec}^2 u$
$y = \sec u$	$y' = u' \cdot \sec u \cdot \operatorname{tg} u$
$y = \operatorname{cosec} u$	$y' = -u' \cdot \operatorname{cosec} u \cdot \operatorname{cotg} u$

FUNCIONES CIRCULARES INVERSAS

Función	Derivada
$y = \operatorname{arc sen} u$	$y' = \frac{u'}{\sqrt{1-u^2}}$
$y = \operatorname{arc cos} u$	$y' = -\frac{u'}{\sqrt{1-u^2}}$
$y = \operatorname{arc tg} u$	$y' = \frac{u'}{1+u^2}$
$y = \operatorname{arc cotg} u$	$y' = -\frac{u'}{1+u^2}$
$y = \operatorname{arc sec} u$	$y' = \frac{u'}{u\sqrt{u^2-1}}$
$y = \operatorname{arc cosec} u$	$y' = -\frac{u'}{u\sqrt{u^2-1}}$

1. Derivar

a) $y = \sqrt{1 - \sqrt{1+x}}$

$$y = \sqrt{x} \rightarrow y' = \frac{1}{2\sqrt{x}}\,dx$$

$$y' = \frac{1}{2\sqrt{1-\sqrt{1+x}}} \cdot \left[\cancel{0} - \frac{1}{2\sqrt{1+x}} \cdot (0+1)\right]$$

$$y' = \frac{1}{2\sqrt{1-\sqrt{1+x}}} \cdot \left[-\frac{1}{2\sqrt{1+x}}\right]$$

$$y' = \frac{-1}{4 \cdot \sqrt{1-\sqrt{1+x}} \cdot \sqrt{1+x}}$$

b) $y = xe^{-x}$

$g(x)$ $f(x)$

$$y = g(x) \cdot f(x) \xrightarrow{y'} g'(x) \cdot f(x) + f'(x) \cdot g(x)$$

$$y' = 1 \cdot e^{-x} + e^{-x}(-1) \cdot x$$

$$y' = e^{-x} - x \cdot e^{-x}$$

c) $y = e^{cotgx^3}$

$$e^{\operatorname{cotg} x^3} = \boxed{e^{\frac{\cos x^3}{\operatorname{sen} x^3}}}$$

$$\operatorname{cotg}(x^3) \neq \operatorname{cotg}^3(x)$$

$$y = \frac{g(x)}{f(x)} \xrightarrow{y'} \frac{g'(x) \cdot f(x) - f'(x) \cdot g(x)}{[f(x)]^2}$$

$$y' = e^{\frac{\cos x^3}{\operatorname{sen} x^3}} \cdot \left[\frac{-\operatorname{sen}(x^3) \cdot 3x^2 \cdot \operatorname{sen}(x^3) - \cos(x^3) \cdot 3x^2 \cdot \cos(x^3)}{\operatorname{sen}^2(x^3)}\right]$$

$$y' = \left[\frac{-3x^2 \overbrace{\left(\operatorname{sen}^2(x^3) + \cos^2(x^3)\right)}^{1}}{\quad}\right] \cdot e^{\frac{\cos x^3}{\operatorname{sen} x^3}}$$

$$\operatorname{sen}^2(x) + \cos^2(x) = 1$$

$$y' = \left[\frac{-3x^2\overbrace{\left(\operatorname{sen}^2(x^3) + \cos^2(x^3)\right)}^{1}}{\operatorname{sen}^2(x^3)}\right] \cdot e^{\frac{\cos x^3}{\operatorname{sen} x^3}}$$

$\operatorname{sen}^2(x) + \cos^2(x) = 1$

$$y' = \frac{-3x^2}{\operatorname{sen}^2(x^3)} \cdot e^{\frac{\cos x^3}{\operatorname{sen} x^3}}$$

✓

d) $y = 5^{\sqrt{4+x^2}}$

e) $y = \ln(\frac{e^{3x}+1}{e^{3x}-1})$

$y = \ln x \xrightarrow{y'} \frac{1}{x}$

$y = \ln(e^{3x}+1) - \ln(e^{3x}-1)$

$$\to y' = \frac{1}{e^{3x}+1} \cdot (e^{3x} \cdot 3 + 0) - \frac{1}{e^{3x}-1} \cdot (e^{3x} \cdot 3)$$

$$y' = \frac{3e^{3x}}{(e^{3x}+1)} - \frac{3e^{3x}}{(e^{3x}-1)}$$

✓

f) $y = e^{x\ln(x)}$

Tarea

✓

g) $\underbrace{g(x) = \ln\left(\sqrt{1+e^x} - 1\right) - \ln\left(\sqrt{1+e^x} + 1\right)}$. Verificar que $g'(x) = \frac{1}{\sqrt{1+e^x}}$

$$y' = \frac{1}{(\sqrt{1+e^x} - 1)} \left(\frac{1}{2\sqrt{1+e^x}} \cdot (0+e^x) - 0\right) - \frac{1}{\sqrt{1+e^x}+1} \cdot \left(\frac{1}{2\sqrt{1+e^x}} \cdot (0+e^x) + 0\right)$$

$$y' = \frac{1}{(\sqrt{1+e^x}-1)}\left(\frac{1}{2\sqrt{1+e^x}}\cdot(0+e^x) - 0\right) - \frac{1}{\sqrt{1+e^x}+1}\cdot\left(\frac{1}{2\sqrt{1+e^x}}\cdot(0+e^x)+0\right)$$

$$y' = \boxed{\frac{e^x}{(\sqrt{1+e^x}-1)\,2\sqrt{1+e^x}}} - \boxed{\frac{e^x}{(\sqrt{1+e^x}+1)(2\sqrt{1+e^x})}}$$

$$y' = \frac{e^x\cdot(\sqrt{1+e^x}+1) - e^x(\sqrt{1+e^x}-1)}{2\sqrt{1+e^x}\cdot(\sqrt{1+e^x}-1)(\sqrt{1+e^x}+1)} \rightarrow a^2-b^2 = (a-b)(a+b)$$

$$y' = \frac{\cancel{e^x\cdot\sqrt{1+e^x}} + e^x \cancel{- e^x\cdot\sqrt{1+e^x}} + e^x}{2\sqrt{1+e^x}\cdot(\cancel{1}+e^x \cancel{-1})}$$

$$y' = \frac{\cancel{2e^x}}{\cancel{2}\sqrt{1+e^x}\cdot\cancel{e^x}} \Rightarrow y' = \frac{1}{\sqrt{1+e^x}}$$

$\therefore$ Se cumple la derivación

h) Sea $y = \ln\left(\sqrt{x+\sqrt{x^2+3}}\right)$. Comprobar que:

$$\frac{4(x^2+3)}{x}y'' + 2y' + \frac{y}{\sqrt{x^2+3}\cdot ln\sqrt{x+\sqrt{x^2+3}}} = 0$$

$$y' = \frac{1}{\sqrt{x+\sqrt{x^2+3}}}\cdot\left(\frac{1}{2\sqrt{x+\sqrt{x^2+3}}}\cdot\left(1+\underbrace{\frac{1}{2\sqrt{x^2+3}}\cdot(2x+0)}\right)\right)$$

$$y' = \frac{1}{\sqrt{x+\sqrt{x^2+3}}\cdot 2\sqrt{x+\sqrt{x^2+3}}}\cdot\left(1+\frac{2x}{2\sqrt{x^2+3}}\right)$$

$$y' = \frac{1}{2(x+\sqrt{x^2+3})}\cdot\left(\frac{2\sqrt{x^2+3}+2x}{2\sqrt{x^2+3}}\right)$$

$$y' = \frac{1}{2\cancel{(x+\sqrt{x^2+3})}}\cdot\frac{\cancel{2}\cancel{(x+\sqrt{x^2+3})}}{\cancel{2}\sqrt{x^2+3}}$$

$$y' = \frac{1}{2\sqrt{x^2+3}} = \boxed{\frac{1}{2}\cdot(x^2+3)^{-1/2}}$$

$$y'' = \frac{1}{2}\cdot\left(-\frac{1}{2}\right)(x^2+3)^{-\frac{1}{2}-1}\cdot(2x+0)$$

$$y'' = \frac{1}{2}\cdot\left(-\frac{1}{2}\right)(x^2+3)^{-\frac{3}{2}}\cdot(2x+0)$$

$$y'' = -\frac{1}{\cancel{4}}\cdot(x^2+3)^{-\frac{3}{2}}\cdot\cancel{2}x$$

$$y'' = \frac{-x}{2\sqrt{(x^2+3)^3}}$$

$$ecc: \frac{4(x^2+3)}{x}\cdot y'' + 2y' + \frac{y}{\sqrt{x^2+3}\cdot\ln\sqrt{x+\sqrt{x^2+3}}} = 0$$

$$\frac{4(x^2+3)}{x}\cdot\left[\frac{-x}{2\sqrt{(x^2+3)^3}}\right] + \cancel{2}\cdot\frac{1}{\cancel{2}\sqrt{x^2+3}} + \frac{\cancel{\ln(\sqrt{x+\sqrt{x^2+3}})}}{\sqrt{x^2+3}\cdot\cancel{\ln(\sqrt{x+\sqrt{x^2+3}})}}$$

$$= \frac{-4\boxed{(x^2+3)^1 x}}{2x\,\boxed{(x^2+3)^{3/2}}} + \underbrace{\frac{1}{\sqrt{x^2+3}} + \frac{1}{\sqrt{x^2+3}}}$$

$$1-\frac{3}{2} = \frac{-1}{2}$$

$$= \frac{-\cancel{4}^{2}\cancel{x}}{\cancel{2x}\cdot\sqrt{x^2+3}} + \frac{2}{\sqrt{x^2+3}} = \frac{-2}{\sqrt{x^2+3}} + \frac{2}{\sqrt{x^2+3}} = 0 \therefore$$ cumple la ecc.