Universidad de Tarapacá
Facultad de Ciencias
Dpto. de Matemática

TUTORIA N°10
CÁLCULO 1

1. **(Ej. Control)** Calcular:

$$\lim_{x\to+\infty}[3-2\frac{ax+1}{ax}]^{\sqrt{x+1}} = \left[3-2\frac{a}{a}\right]^{\sqrt{\infty+1}}$$

$$= [1]^{\infty} \to F.I$$

lim espec

$$\lim_{x\to\infty}\left[1+\frac{1}{x}\right]^{x} = e$$

p/ desarrollar

$$\lim_{x\to+\infty}\left[1+\left[\frac{2}{1}-2\cdot\frac{ax+1}{ax}\right]\right]^{\sqrt{x+1}} = \lim_{x\to+\infty}\left[1+\left(\frac{2(ax)-2(ax+1)}{ax}\right)\right]^{\sqrt{x+1}}$$

$$= \lim_{x\to+\infty}\left[1+\frac{\cancel{2ax}-\cancel{2ax}-2}{ax}\right]^{\sqrt{x+1}} = \lim_{x\to+\infty}\left[1+\left(\frac{-2}{ax}\right)\right]^{\sqrt{x+1}\cdot\left(\frac{ax}{-2}\cdot\frac{-2}{ax}\right)}$$

$$= \lim_{x\to+\infty}\left(\left[1+\left(\frac{-2}{ax}\right)\right]^{\frac{ax}{-2}}\right)^{\sqrt{x+1}\cdot\left(\frac{-2}{ax}\right)}$$

lim esp.

$$= \left(\underbrace{\left[\lim_{x\to+\infty}1+\left(\frac{-2}{ax}\right)\right]^{\frac{ax}{-2}}}_{e}\right)^{\sqrt{x+1}\cdot\left(\frac{-2}{ax}\right)} = e^{\lim_{x\to+\infty}\frac{-2}{ax}\cdot\sqrt{x+1}} \to = 0$$

$$;\ \lim_{x\to+\infty}\frac{-2}{ax}\cdot\sqrt{x+1}$$

$$\lim_{x\to+\infty}\frac{-2}{ax}\cdot\sqrt{x^2\cdot\left(\frac{1}{x}+\frac{1}{x^2}\right)} \to \sqrt{x+1}$$

$$\sqrt{x^2}=|x| = \begin{cases} x, & x\geq 0 \\ -x, & x<0 \end{cases} \quad x$$

$$\sqrt{4} = \sqrt{2^2} = 2$$

$$= \lim_{x\to+\infty}\frac{-2}{a\cancel{x}}\cdot\cancel{x}\sqrt{\frac{1}{x}+\frac{1}{x^2}}$$

$$= \lim_{x\to+\infty}\underbrace{\frac{-2}{a}}_{cte}\cdot\sqrt{\frac{1}{x}+\frac{1}{x^2}} = \frac{-2}{a}\cdot\sqrt{\underbrace{\lim_{x\to+\infty}\left(\cancel{\frac{1}{x}}^{0}+\cancel{\frac{1}{x^2}}^{0}\right)}_{0}} \qquad \lim_{x\to\infty}\frac{1}{x^n}=0$$

$$= -\frac{2}{a}\cdot\sqrt{0} = 0$$

$$= e^{0}$$

$$= 1$$

$$\therefore \lim_{x\to+\infty}\left(3-2\cdot\frac{ax+1}{ax}\right)^{\sqrt{x+1}} = 1$$

2. Determinar A y B de modo que f sea continua en todo su Dominio

$$f(x) = \begin{cases} -2sen(x), x \le -\frac{\pi}{2} \\ Asen(x) + B\,, -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \cos(x)\,, x \ge \frac{\pi}{2} \end{cases}$$

Rsp:

$A = -1$

$B = 1$

3. **(Ej. Prueba)** Sea las siguientes funciones calcular sus asíntotas

a) $f(x) = \begin{cases} x + 1 + \frac{1}{x+1}, si\ x < -1 \\ \frac{2x^2}{x^2+1}, si\ x > -1 \end{cases}$ $(-\infty, -1)$ $(-1, +\infty)$

Izq. Derech. $+$

i) $\text{Dom } f = \mathbb{R} - \{-1\} = (-\infty, -1) \cup (-1, +\infty)$ Derecha

$-\infty$ -1 $-0{,}995$ $-1{,}005$

A.V: $\lim\limits_{x \to -1^-} \left[(x+1) + \frac{1}{x+1}\right]$

$\lim\limits_{x \to -1^-} \left[\frac{(x+1)^2+1}{x+1}\right] = \frac{\overbrace{(-1{,}005+1)^2}^{-} + 1}{\underbrace{-1{,}005+1}_{-}} = \frac{+}{-} = -\infty$

$\lim\limits_{x \to -1^+} \frac{2x^2}{x^2+1} = \frac{2(-0{,}995)^2}{(-0{,}995)^2+1} = \frac{+}{+} = +\infty$

$\therefore$ $x = -1$ es una Asíntota vertical.

A.O.D: ① $\lim\limits_{x \to +\infty} \frac{f(x)}{x} = m$ $\wedge$ ⓘⓘ $\lim\limits_{x \to +\infty} [f(x) - m \cdot x] = b$

① $\lim\limits_{x \to +\infty} \underbrace{\left(\frac{2x^2}{x^2+1}\right)}_{2} \cdot \underbrace{\frac{1}{x}}_{0} = \underbrace{\left[\lim\limits_{x \to +\infty} \frac{2x^2}{x^2+1}\right]}_{2} \cdot \underbrace{\left[\lim\limits_{x \to +\infty} \frac{1}{x}\right]}_{0}$

$= 2 \cdot 0 = \boxed{0 = m}$

ⓘⓘ $\lim\limits_{x \to +\infty} \left[\frac{2x^2}{x^2+1} - 0\right] = \lim\limits_{x \to +\infty} \frac{2x^2}{x^2+1} = \boxed{2 = b}$

$\therefore$ No hay A.O.D y $y = 2$ es una A. Horizontal.

A.O.I: ① $\lim$ $\frac{f(x)}{x} \to f(x) \cdot \frac{1}{x}$ $= m$ $\wedge$ ⓘⓘ $\lim$ $[f(x) - m \cdot x] = 0$

$$A.O.I: \ (i) \lim_{x\to-\infty} \frac{f(x)}{x} = m \ \wedge \ (ii) \lim_{x\to+\infty} [f(x) - m\cdot x] = 0$$

$f(x)\cdot\frac{1}{x}$

$$(i) \lim_{x\to-\infty} \left(x+1+\frac{1}{x+1}\right)\cdot\frac{1}{x} = \lim_{x\to-\infty} \left(\frac{(x+1)^2+1}{x+1}\right)\cdot\frac{1}{x} = \lim_{x\to-\infty} \frac{x^2+2x+1+1}{x^2+x} \quad \left(\frac{1}{x^2}\right)$$

$$\lim_{x\to\infty} \frac{1}{x^n} = 0$$

$$= \lim_{x\to-\infty} \frac{1+\frac{2}{x}+\frac{2}{x^2}}{1+\frac{1}{x}} = \frac{\lim_{x\to-\infty} 1 + \cancel{\lim_{x\to-\infty} \frac{2}{x}}^{0} + \cancel{\lim_{x\to-\infty}\frac{2}{x^2}}^{0}}{\lim_{x\to-\infty} 1 + \cancel{\lim_{x\to-\infty}\frac{1}{x}}^{0}} = \frac{1}{1} = \boxed{1 = m}$$

$$(ii) \lim_{x\to-\infty} \left[\underbrace{\left(x+1+\frac{1}{x+1}\right)}_{\text{Desarrollar}} - x\right] = \lim_{x\to-\infty} \left[\frac{x^2+2x+2}{x+1} - x\right]$$

$$= \lim_{x\to-\infty} \frac{x^2+2x+2-x(x+1)}{x+1}$$

$$= \lim_{x\to-\infty} \frac{\cancel{x^2}+2x+2-\cancel{x^2}-x}{x+1} = \lim_{x\to-\infty} \frac{x+2}{x+1} = 1 = b$$

$y = mx + b$

$\therefore$ Existe A.O.I : $y = x+1$ ///

b) $f(x) = \dfrac{3x^2-2+sen(x)}{x+1}$ $\quad$ Dom f = $\mathbb{R}-\{-1\}$ ✓

$\mathbb{R}$ ... $\to \mathbb{R}-\{-1\}$

$\to \boxed{(-\infty,-1)\cup(-1,+\infty)}$

Izq $\quad$ Derech

$-1{,}005 \quad -1 \quad -0{,}995$

cálculo por calculadora

A.V. $\displaystyle\lim_{x\to-1^-} \frac{3x^2-2+\operatorname{sen} x}{x+1}$

$$= \lim_{x\to-1^-} \frac{\overset{+}{3(-1{,}005)^2-2} + \overbrace{\operatorname{sen}(-1{,}005)}^{0{,}0175}}{-1{,}005+1} = \frac{1{,}030075-0{,}0175}{-0{,}005} = \frac{+}{-} = -\infty$$

$$= \lim_{x\to-1^+} \frac{3x^2-2+\operatorname{sen} x}{x+1}$$

$$= \frac{3(-0{,}995)^2-2+\overbrace{\operatorname{sen}(-0{,}995)}}{-0{,}995+1} = \frac{2{,}970-2-\operatorname{sen}(0{,}98)}{0{,}005} = \frac{\overbrace{0{,}952}^{+}}{\underbrace{0{,}005}_{+}} = +\infty$$

$$A.O.D: \ (i) \lim_{x\to+\infty} \frac{f(x)}{x} = m \ \wedge \ \lim_{x\to-\infty} [f(x) - mx] = b$$

$$(i) \lim_{x\to+\infty} \frac{3x^2-2+\operatorname{sen} x}{x+1}\cdot\frac{1}{x} = \lim_{x\to+\infty} \frac{3x^2-2+\operatorname{sen} x}{x^2+x} = \boxed{3 = m}$$

(i) $\lim_{x\to+\infty} \frac{3x^2-2+\operatorname{sen} x}{x+1}\cdot\frac{1}{x} = \lim_{x\to+\infty}\frac{3x^2-2+\operatorname{sen} x}{x^2+x} = \boxed{3 = m}$

(ii) $\lim_{x\to+\infty}\left[\frac{3x^2-2+\operatorname{sen} x}{x+1} - 3\cdot x\right]$

$= \lim_{x\to+\infty}\left[\frac{3x^2-2+\operatorname{sen} x-3x(x+1)}{x+1}\right] = \lim_{x\to+\infty}\left[\frac{\cancel{3x^2}-2+\operatorname{sen} x-\cancel{3x^2}-3x}{x+1}\right]$

$= \lim_{x\to+\infty}\left[\frac{-3x-2+\operatorname{sen} x}{x+1}\right] = \boxed{-3 = b}$

$\therefore$ $y = 3x-3$ es una Asíntota Oblicua Derecha

A.O.I. (i) $\lim_{x\to-\infty}\frac{f(x)}{x} = m \wedge \lim_{x\to-\infty}[f(x)-mx] = b$

(i) $= \lim_{x\to-\infty}\frac{3x^2-2+\operatorname{sen} x}{x+1}\cdot\frac{1}{x} = \lim_{x\to-\infty}\frac{3x^2-2+\operatorname{sen} x}{x^2+x} = 3 = m$

(ii) $\lim_{x\to-\infty}\frac{3x^2-2+\operatorname{sen} x}{x+1} - 3x = \lim_{x\to-\infty}\frac{3x^2-2+\operatorname{sen} x-3x(x+1)}{x+1}$

$= \lim_{x\to-\infty}\frac{\cancel{3x^2}-2+\operatorname{sen} x-\cancel{3x^2}-3x}{x+1} = \lim_{x\to-\infty}\frac{-3x-2+\operatorname{sen} x}{x+1}$

$= \boxed{-3 = b}$ $\therefore$ $y = 3x-3$ es una A.O.I

c) $\lim_{x\to1}\frac{e^{x-1}+sen(x-1)-1}{\ln(x)}$ $= \frac{e^{1-1}+\operatorname{sen}(1-1)-1}{\ln(1)} = \frac{1+0-1}{0} = \frac{0}{0}$

$\lim_{x\to0}\frac{e^x-1}{x} = 1$

$\lim_{x\to0}\frac{\operatorname{sen} x}{x} = 1$

Sea $x-1 = t$ $\quad x\to1$, $t\to0$

$1-1 = t$

$0 = t$

$\to x = t+1$

$= \lim_{t\to0}\frac{e^t+\operatorname{sen}(t)-1}{\ln(t+1)}$

$= \lim_{t\to0}\frac{(e^t-1)+\operatorname{sen}(t)}{\ln(t+1)\cdot\frac{t}{t}}$ 1

$= \lim \ (e^t-1)+\operatorname{sen} t$

$$t \to 0 \qquad \ln(t+1)\cdot\left(\frac{t}{t}\right) 1$$

$$= \lim_{t\to 0} \frac{(e^{t}-1)+\operatorname{sen} t}{t\cdot\frac{1}{t}\ln(t+1)}$$

$$= \lim_{t\to 0}\left[\frac{(e^{t}-1)+\operatorname{sen} t}{t}\cdot\frac{1}{\frac{1}{t}\cdot\ln(t+1)}\right]$$

$$= \left[\underbrace{\lim_{t\to 0}\frac{e^{t}-1}{t}}_{1} + \underbrace{\lim_{t\to 0}\frac{\operatorname{sen} t}{t}}_{1}\right]\cdot\frac{\lim_{t\to 0} 1}{\lim_{t\to 0}\frac{1}{t}\ln(t+1)}$$

$(*)$

$$= 2\cdot\frac{1}{\lim_{t\to 0}\frac{1}{t}\cdot\ln(t+1)}$$

$$\lim_{t\to 0}\frac{1}{t}\cdot\ln(t+1) = \lim_{t\to 0}\ln(t+1)^{1/t}$$

$$\lim_{x\to 0}(1+x)^{1/x} \ne \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}$$

$$= \ln \underbrace{\lim_{t\to 0}(t+1)^{1/t}}_{e}$$

$$= \ln e = 1$$

Continuación $(*)$

$$= 2\cdot\frac{1}{1} = 2$$

$$\therefore \lim_{x\to 1}\frac{e^{x-1}+\operatorname{sen}(x-1)-1}{\ln x} = 2$$