Universidad de Tarapacá
Facultad de Ciencias
Dpto. de Matemática

TUTORIA N°5
CÁLCULO 1

1. EJERCICIOS DE ALGEBRA DE FUNCIONES

a) Sean $f(x) = 3x - 20$ y $g(x) = \begin{cases} 2x, si\ 0 \leq x < 1 \to g_1 \\ \frac{2}{x}, si\ 1 \leq x < 4 \to g_2 \\ 3, si\ 4 \leq x \to g_3 \end{cases}$ Determinar $\cancel{\frac{g}{f}}$ $\frac{f}{g}$

i) Combinaciones:

$$\frac{f}{g_1} = \frac{f(x)}{g_1(x)} = \frac{3x-20}{2x} \to Dom\ \frac{f}{g_1} = Dom\ f \cap Dom\ g_1 - \{x \in \mathbb{R} / g_1(x) = 0\}$$

$$= \mathbb{R} \cap [0,1) - \{g_1(x) = 0\}$$

$$= \mathbb{R} \cap (0,1)$$

$$Dom\ \frac{f}{g_1} = (0,1) \#$$

$$\frac{f}{g_2} = \frac{f(x)}{g_2(x)} = \frac{3x-20}{\frac{2}{x}} = \frac{3x^2}{2} - 10x \quad Dom\ \frac{f}{g_2} = Dom\ f \cap Dom\ g_2$$

$$= \mathbb{R} \cap [1,4)$$

$$Dom\ \frac{f}{g_2} = [1,4)$$

$$\frac{f}{g_3} = \frac{f(x)}{g_3(x)} = \frac{3x-20}{3} \to Dom\ \frac{f}{g_3} = Dom\ f \cap Dom\ g_3$$

$$= \mathbb{R} \cap [4,+\infty)$$

$$Dom\ \frac{f}{g_3} = [4,+\infty) \#$$

$$\therefore \left(\frac{f}{g}\right)(x) \begin{cases} \frac{3x-20}{2x}, & x \in (0,1) \\ \frac{3}{2}x^2 - 10, & x \in [1,4) \\ x - \frac{20}{3}, & x \in [4,+\infty) \end{cases}$$

b) Sean $f(x) = \begin{cases} x^2 - 1, si\ x \in (0,3) \to f_1 \\ 4 - 3x, si\ x \in (-3,0] \to f_2 \end{cases}$ y $g(x) = -3$, Determinar $f + g$

$Dom\ g = \mathbb{R}$

$4-3x, si\ x \in (-3,0]$ $\to f_2$

$Dom\ g = \mathbb{R}$

i) Combinac.

$(f_1+g)(x) = f_1(x)+g(x)$
$= x^2-1+(-3)$
$(f_1+g)(x) = x^2-4 \longrightarrow Dom(f_1+g) = Dom f_1 \cap Dom g$
$= (0,3) \cap \mathbb{R}$
$Dom(f_1+g) = (0,3)$

$(f_2+g)(x) = f_2(x)+g(x)$
$= 4-3x+-3$
$= 1-3x \longrightarrow Dom(f_2+g) = Dom f_2 \cap Dom g$
$= (-3,0] \cap \mathbb{R}$
$Dom(f_2+g) = (-3,0]$

$$\therefore (f+g)(x) = \begin{cases} x^2-4, & x \in (0,3) \\ 1-3x, & x \in (-3,0] \end{cases}$$

c) **(Ej. Prueba)** Sean $f(x) = \sqrt{|x^2+1|+2}$ siendo $x \in [-3,3)$ y $g(x) = \sqrt{\frac{|x^2+1|-2}{x^2-6x+14}}$, Determinar $\frac{f}{g}$

i) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \longrightarrow Dom(f/g) = Dom f \cap Dom g - \{x \in \mathbb{R} / g(x) = 0\}$

$Dom\ f = [-3,3)$

$Dom\ g = \{x \in \mathbb{R} : g(x) \in \mathbb{R}\}$
$= \{\mathbb{R} : \sqrt{\frac{|x^2+1|-2}{x^2-6x+14}} \in \mathbb{R}\}$
$= \{\mathbb{R} : \frac{|x^2+1|-2}{x^2-6x+14} \geq 0\}$
$= \{\mathbb{R} : \frac{|x^2+1|-2}{x^2-6x+14} \geq 0,\ x^2-6x+14 > 0$
$x^2-6x+9-9+14 > 0$
$(x-3)^2+5 > 0$
$\Rightarrow \forall x \in \mathbb{R}$

$= \{\mathbb{R} : |x^2+1|-2 \geq 0,\ (x-3)^2+5\ \forall x \in \mathbb{R}\}$
$= \{\mathbb{R} : |x^2+1| \geq 2,\ (x-3)^2+5\ \forall x \in \mathbb{R}\}$
$= \{\mathbb{R} : x^2+1-2 \geq 0,$ " " $\}$
$= \{\mathbb{R} : x^2-1 \geq 0,\ \mathbb{R}$
$= \{\mathbb{R} : (-\infty,-1] \cup [1,+\infty),\ \mathbb{R}\ \}$

$= \{\mathbb{R} : x^2 - 1 \geq 0 \ , \ \mathbb{R}\}$

$= \{\mathbb{R} : (-\infty; -1] \cup [1,+\infty) \ , \ \mathbb{R}\}$

$= \{(-\infty,-1] \cup [1,+\infty)$

$\text{Dom}\, g = (-\infty,-1) \cup (1,+\infty)$

$\therefore \text{Dom}\, f/g = [-3,3) \cap [(-\infty,-1) \cup (1,+\infty)]$

$\text{Dom}(f/g) = [-3,-1) \cup (1,3)$

$\therefore$ Exp. Analítica de $(f/g)(x) = \dfrac{f(x)}{g(x)} = \dfrac{\sqrt{|x^2+1|+2}}{\sqrt{\dfrac{|x^2+1|+2}{x^2-6x+14}}}, \ x \in [-3,-1) \cup (1,3)$

1. EJERCICIOS DE COMPOSICION DE FUNCIONES

a) Sean $g(x) = \sqrt{x}$ y $f(x) = 2x - 3$, Determinar fog

$\text{Dom}(f \circ g) = \{x \in \text{Dom}\, g \ \wedge g \in \text{Dom}\, f\}$

$\text{Dom}\, f = \mathbb{R}$ $\quad = \{x \in \mathbb{R}_0^+ \wedge \sqrt{x} \in \mathbb{R}\}$ $\quad \hookrightarrow x \geq 0$

$\text{Dom}\, g = \mathbb{R}_0^+$ $\quad = \{x \in \mathbb{R}_0^+ \wedge x \in [0,+\infty)\}$

$\text{Dom}(f \circ g) = \mathbb{R}_0^+$

$\therefore \exists (f \circ g)(x) = f(g(x))$

$= f(\sqrt{x})$

$= 2\sqrt{x} - 3, \ x \in \mathbb{R}_0^+$

b) Sean $g(x) = \sqrt{x}$ y $f(x) = 2x - 3$, Determinar gof

$\text{Dom}\, g \circ f = \{x \in \text{Dom}\, f \ \wedge f \in \text{Dom}\, g\}$

$\text{Dom}\, f = \mathbb{R}$ $\quad = \{x \in \mathbb{R} \wedge 2x - 3 \in \mathbb{R}_0^+\}$

$\text{Dom}\, g = \mathbb{R}_0^+$ $\quad = \{x \in \mathbb{R} \wedge 2x - 3 \geq 0\}$

$= \{x \in \mathbb{R} \wedge 2x \geq 3\}$

$= \{x \in \mathbb{R} \wedge x \geq \frac{3}{2}\}$

$= \{ x \in \mathbb{R} \wedge x \geq \frac{3}{2} \}$

$\text{Dom } g\circ f = [\frac{3}{2}, +\infty)$

$\therefore \exists (g\circ f)(x) = g(f(x)) = g(2x-3)$
$= \sqrt{2x-3}, \quad x \in [\frac{3}{2}, +\infty)$

c) **(Ej. Prueba)** Sean $f(x)\begin{cases} \frac{1}{x-1}, si\ x \in (-1,1) & \to f_1 \\ |x^2+1|, si\ x \in (1,2) & \to f_2 \end{cases}$ y $g(x)\begin{cases} [x], si\ x \in [0,1) & \to g_1 \\ \sqrt{x^2-1}\ si\ x \in [1,3] & \to g_2 \end{cases}$ $f\circ g$

Def P.Entera $m \leq [\ ] < m+1$

$[x], x \in [0\ 1)$

$0 \leq [\ ] < 1 \Rightarrow 0, \ x \in [0,1)$

$g(x) = \begin{cases} 0, & x \in [0,1) \to g_1 \\ \sqrt{x^2-1}, & x \in [1\ 3) \to g_2 \end{cases}$

i) Combinaciones

$\text{Dom } f_1\circ g_1 = \{ x \in \text{Dom } g_1 \wedge g_1 \in \text{Dom } f_1 \}$
$= \{ x \in [0,1) \wedge 0 \in (-1,1) \}$
$= \{ x \in [0,1) \wedge \text{Verdad} \}$
$= [0,1)$

$\therefore \exists (f_1\circ g_1)(x) = f(g_1(x)) = f_1(0) = -1, \ x \in [0,1)$

$\text{Dom } f_1\circ g_2 = \{ x \in \text{Dom } g_2 \wedge g_2 \in \text{Dom } f_1 \}$
$= \{ x \in [1,3) \wedge \sqrt{x^2-1} \in (-1,1) \}$
$= \{ x \in [1\ 3) \wedge -1 < \sqrt{x^2-1} < 1 \}$ $\to \sqrt{\ } \geq 0$
$= \{ x \in [1,3) \wedge 0 \leq \sqrt{x^2-1} < 1 \ /(\ )^2 \}$
$= \{ x \in [1,3) \wedge 0 \leq \sqrt{x^2-1} < 1 \}$
$= \{ x \in [1,3) \wedge [0 \leq x^2-1 \wedge x^2-1 < 1 /(+1)]$

$= \{ x \in [1,3) \wedge [0 \leq x^2 - 1 \wedge x^2 - 1 < 1 / (+1)] \}$

$= \{ x \in [1,3) \wedge [1 \leq x^2 \wedge x^2 < 2 / \sqrt{\ }] \}$

$= \{ x \in [1,3) \wedge [1 \leq x \wedge x < \sqrt{2}] \}$

$= \{ x \in [1,3) \wedge x \in [1, \sqrt{2}) \}$

$= [1, \sqrt{2})$ #

$\therefore \exists (f_1 \circ g_2)(x) = f_1(\sqrt{x^2-1}) = \dfrac{1}{\sqrt{x^2-1}-1}, \; x \in [1, \sqrt{2})$

$\mathrm{Dom}\, f_2 \circ g_1 = x \in \mathrm{Dom}\, g_1 \wedge g_1 \in \mathrm{Dom}\, f_2$ ✓

$= x \in [0,1) \wedge 0 \in (1,2)$

$= x \in [0,1) \wedge$ Falsedad

$\mathrm{Dom}\, f_2 \circ g_1 = \emptyset$

$\therefore \nexists f_2 \circ g_1$

$\mathrm{Dom}\, f_2 \circ g_2 = \{ x \in \mathrm{Dom}\, g_2 \wedge g_2 \in \mathrm{Dom}\, f_2 \}$ ✓

$= \{ x \in [1,3) \wedge \sqrt{x^2-1} \in (1,2) \}$

$= \{ \;'' \wedge 1 < \sqrt{x^2-1} < 2 / (\ )^2 \}$

$= \{ \;'' \wedge 1 < |x^2-1| < 4 \}$ (+)

$= \{ \;'' \wedge 1 < x^2 - 1 < 4 / (+1) \}$

$= \{ \;'' \wedge 2 < x^2 < 5$ ✓

$= \{ \;'' \wedge [2 < x^2 \wedge x^2 < 5]$

$= \{ \;'' \wedge [x^2 - 2 > 0 \wedge x^2 - 5 < 0]$

$(x-\sqrt{2})(x+\sqrt{2}) > 0 \qquad (x-\sqrt{5})(x+\sqrt{5}) < 0$

$= \{ \;'' \wedge [(-\infty, -\sqrt{2}) \cup (\sqrt{2}, +\infty) \wedge (-\sqrt{5}, \sqrt{5})]$

$= \{ \;'' \wedge [(-\sqrt{5}, -\sqrt{2}) \cup (\sqrt{2}, \sqrt{5})]$

$[1,3) \wedge$

$= (\sqrt{2}, \sqrt{5})$ ✓

$$\therefore \exists (f_2 \circ g_2)(x) = f\left(\sqrt{x^2-1}\right) = \left|\left(\sqrt{x^2-1}\right)^2+1\right| = x^2, \; x \in \left(\sqrt{2}, \sqrt{5}\right)$$

$$\therefore f \circ g(x) = \begin{cases} -1, & x \in [0,1) \\ \dfrac{1}{\sqrt{x^2-1}-1}, & x \in [1, \sqrt{2}) \\ x^2, & x \in \left(\sqrt{2}, \sqrt{5}\right) \end{cases}$$